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A Study and Comparison of Different 3D Reconstruction Methods Following Quality Criteria

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Abstract

3D image of a real object is a process that must be passed through two stages. The first is scanning real object by using 3D scanner, this step allows the acquisition of 3D point cloud of the object. The second is the reconstruction step, where the construction of the mesh that represents the real object is done. The surface reconstruction is carried out by means of an existing surface reconstruction method. Mesh reconstruction techniques can be grouped into two categories: the combinatorial approach and the approach by adjusting a predefined model. A large number of combinatorial methods have the principle of establishing relations between the points of a sample. The second approach is based on the idea of approximating the sampled surface using predefined models, built on global or local assumptions concerning the shape to be reconstructed. In this paper, a review of literature and experimental studies of 3d reconstruction methods, that exist in the literature, are realized then a comparison, between these methods based on Frey criterion that represents the quality of the produced surface and execution time. The experimental results show that in terms of surface quality, Ball Pivoting technique, presents a good result. However, alpha shapes method gives relevant results in execution time.

Keywords*:* *3D reconstruction, Delaunay triangulation, Alpha Shapes, Ball Pivoting Algorithm, Poisson Method, Frey Quality, RBF, MLS.*

1 Introduction

In the literature, there are many 3D reconstruction methods which present a challenge in the choice of the most suitable method to be used, for this reason it has been proposed to make a comparative study between some reconstruction techniques of different categories. In recent years, tridimensional geometric modeling of real objects, by points cloud resulting from their digitization, has become an important research topic, especially with the development of 3D scanners. Moreover, the need for digital 3D models in many fields of applications, such as, industrial design, medical imaging and virtual environments. There are different approaches to reconstruct mesh from scattered data points. In general, there are two main approaches. The first is based on combinatorial geometry whereas the second is the implicit approach. The main idea of combinatorial method is to establish connectivity relationships between neighboring points on the sampled surface. The Implicit approach uses an implicit representation of shapes defined from a mathematical function from which we extract an iso-surface.

Among the combinatorial reconstruction methods recognized in the literature there is one based on the Delaunay triangulation [1], this method is proposed by B.Delaunay. The Delaunay technique presents a good duality with the Voronoi diagram, this duality is shown by Boissonat [2][3]. There are other combinatorial methods such as Crust method [4], alpha-shape method [5] and the Ball-Pivoting Algorithm (BPA) [6] inspired by the α-shapes model. As For the implicit approach, there are Poisson technique [7] and Oztireli et al. method [8], there are several reconstruction algorithms such as methods based on Radial Basis Functions (RBF) [9][10][11], and there are others based on the Moving Least Squares approximation [12][13].

In this paper we present a review of various reconstruction methods of surface available in the literature. Thus, a comparison between these techniques, based on the quality of the surfaces and the speed of calculation, will be made.

This work is organized as follows: in section 2, we evoke quality of a 3D surface. Then, in section 3, we present different types of reconstruction methods. Afterwards, in section 4, we lay out the experimental results and comparison. Finally, we wrap up with a conclusion.

2 Quality Of a 3d Surface According to Frey

The quality of the surface mesh is an important criterion because of its effect on the precision and the coherence of the numerical results and the convergence of the calculation. The quality Q_T of a triangle T is a numerical value that gives a measure of its geometric shape. By convention, this varies between 0 (flat triangle) and 1(equilateral triangle). There are different metrics to evaluate the quality of mesh, such as compactness according to Guéziec [14], criterion of Frey developed by Frey [15], Normalized Shape Ratio (NSR), Jacobien Ratio and Angle Skew. In this work we will use the criterion of Frey to evaluate mesh.

According to Frey [15] the quality of triangle T is defined as:

$$Q_T = 2\sqrt{3}\frac{r_T}{L_{MAX}} = 2\sqrt{3}\ \frac{S_T}{P_T.L_{MAX}} \tag{1}$$

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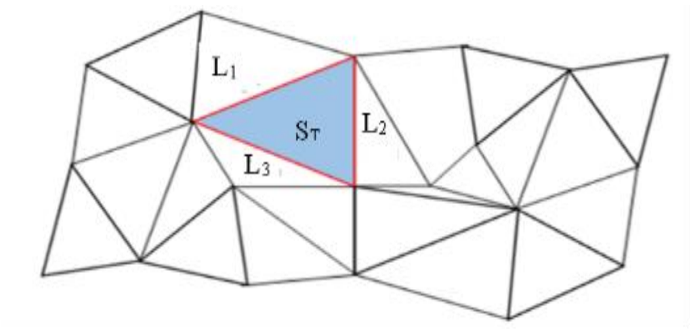


Fig. 1 Surface quality according to Frey

Where, L_{MAX} is the length of the longest side of the triangle T. r_T is the radius of the circle inscribed at T; it is calculated as the ratio of the area S_T by the half-perimeter P_T. A triangle is said to be compact if $Q_T \geq Q_{TS}$ with Q_{TS} has value 0.6 by experimentation. A surface is compact if the percentage of compact triangles is greater than or equal to 50%.

3 Reconstruction Methods

3.1 Delaunay basis methods

In computational geometry and mathematics, the Delaunay triangulation for a scattered point cloud P is a triangulation DT(P) such that no point of P lies inside the circumcircle of any triangle in DT(P). Delaunay triangulation maximizes the minimum of all the angles of the triangles in the triangulation. There are different algorithms for constructing Delaunay triangulation such as Fortune Algorithm [16], Watson algorithm [17] and Lawson algorithm [18]. ‘’Divide and Conquer’’ is used in [19], the main idea of the process that consists in solving the problem by decomposing it into a set of two sub-problems. Firstly, the points are sorted in lexicographical order and the points cloud is successively subdivided into two blocks until we obtain two octrees, from which we construct the Delaunay triangulation. The triangulation is completed by merging the two Delaunay sub-triangulations.

3.1.1 Method of alpha shapes

The concept of alpha shapes is proposed by Edelsbrunner et al. [5], it is a generalization of the convex hull and a subgraph of the Delaunay triangulation, a real parameter α controls desired level of detail. Firstly, we introduce some theoretical concepts related to alpha shape. Let S be a set of points, any subset $T \subset S$, defined by k+1 points with $0 \leq k \leq 3$, defines a k-simplex σ_T which is the convex hull of T. As shown in fig. 2.

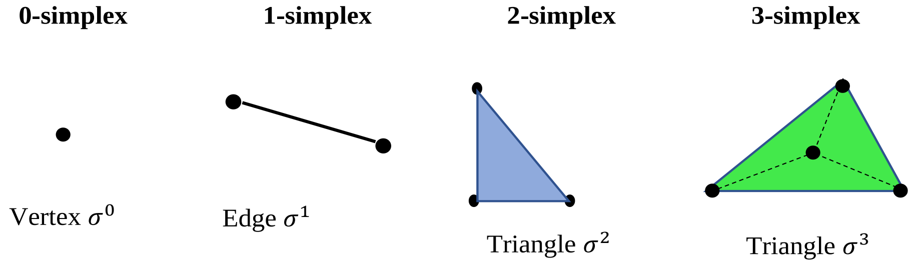


Fig. 2 k-simplex

Simplicial complex is a collection K of k-simplices for $0 \leq k \leq 3$, that satisfies the following two conditions:

1) If σ_T is a simplex in K, all its faces are simplices in K.
2) The intersection of two simplices in K is either $\emptyset$ or a face of both.

The alpha complex C_α is simplicial subcomplex of Delaunay Triangulation $DT(S)$. Each k-simplex $\sigma_T \in$ DT(S) is in the alpha complex C_α if:

1) The circumcircle of σ_T, with radius r_T less than α, is empty.
2) σ_T is a face of another simplex in C_α.

3.1.1.1 Alpha shapes algorithm

- Compute the Delaunay triangulation $DT(S)$ of S,
- Then we determine C_α by inspecting all simplices σ_T in DT(S) whose circumcircle has the radius less than α. This step is called it the alpha test.

The union of all simplices of alpha complex C_α represent the α-shape S_α. Fig. 3 presents a demonstration of alpha shape.

Fig. 3 alpha shape demonstration

3.1.1.2 Alpha shapes limitations

The deficits of the Alpha shape method in surface reconstruction consist in how to choose the value of alpha, and also if the point cloud is not uniformly sampled there is no alpha satisfying or approximating the object surface.

3.1.2 The Ball Pivoting method

The Ball Pivoting Algorithm of Bernardini et al. [6] gets its name from using a virtual ball that pivots to reconstruct a mesh from a set of scattered 3D points. The Ball Pivoting algorithm is composed of two steps but, firstly, some vocabulary related to this method should be highlighted:

- Seed facet: an orphan facet built to serve as starting facet for the triangulation expansion.
- Expansion front: a set of edges from which the triangulation will be expanded.

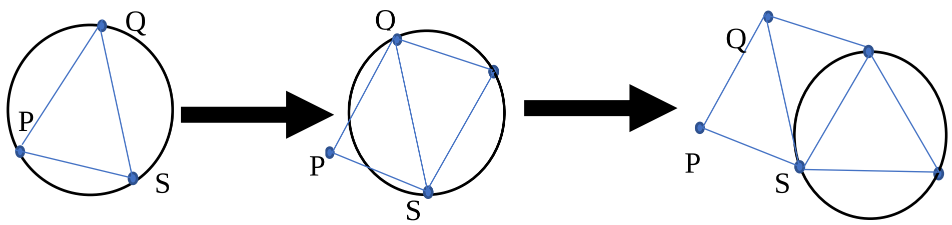


Fig. 4 Presentation of Ball Pivoting Algorithm

3.1.2.1 Ball pivoting algorithm

- **Finding a seed facet** consists of searching for a seed facet by starting with a single point P, and testing, for each set of two neighbours (Q,S), if (P,Q,S) can build a triangle such that the r-ball passing through this triplet of points without containing any other point from data set and checking if the triangle is consistently oriented. Finally, we stop when a seed triangle is found and its three edges are added to the expansion front.
- **Expanding the triangulation,** the ball pivots from each edge in the seed facet, looking for a third point. It pivots until it gets caught in the triangle formed by the edge and the third point. A new triangle is formed, and the algorithm tries to expand from it. This process continues until the ball can't find any point to expand to. At this point, the algorithm looks for a new seed triangle, and the process described above starts all over.

3.1.2.2 Ball pivoting limitations

- The smaller the ball, the more holes and disconnected parts will be created.
- The larger the ball, the more details will be lost.

3.1.3 Crust method

3.1.3.1 Crust algorithm

Crust algorithm was proposed by Amenta [4] which consists of four steps. First, the Voronoi diagram of points is calculated after which the poles of the Voronoi cells are determined, defined as the two vertices of these cells furthest from the generating point of the cell. The third step is building the Delaunay triangulation of the union of the points cloud and the poles defined in the previous step. At the end, only the triangles whose three vertices are points of the starting data are kept. This step is called Voronoi filtering. Yet, these facets do not generally form the final surface as it requires a second filtering process by eliminating the triangles which do not satisfy the normal criterion, which specifies that the normal to the triangles forms small angles with the vectors formed by the vertices of the triangles and their poles. Crust algorithm is described in fig. 5.

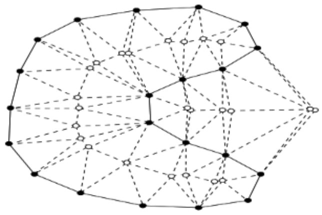


Fig. 5 2D example of the Delaunay triangulation of the sample points (black) and their Voronoi poles (white). The Crust is indicated by solid lines.

3.1.3.2 Crust method limitations

- The point cloud must be dense enough to allow a good reconstruction.
- Disadvantageous method with noisy data.

3.2 Implicit Reconstruction Methods

Implicit reconstruction methods such as Poisson method [7], Hoppe et al. [9] can be based on Radial Basis Functions (RBF) such as [20] [21][22], or on Moving Least Squares (MLS) such as [13] [8] [23] [24]. In this paragraph below we will define some basic concepts.

3.2.1 The implicit surface interpolation

It assumes that we have a set of points cloud, $X = (x_1, x_2, \dots, x_n)$ with $x_i \in \boldsymbol{R}^3$. The implicit surface S is described as the position of points at which the function F takes the value zero such that:

$$S = \{\forall\, x_i \in X / F(x_i) = 0\} \quad (6)$$

The implicit surface interpolation techniques consist of constructing this function F by knowing some information or constraints on F.

3.2.1.1 Radial Basis Functions interpolation

Radial Basis Function (RBF) techniques are now widely used in many subjects like geometric data analysis, neural networks, and the interpolation scattered data. ψ denotes a RBF. The issue of scattered data interpolation can be stated as follows: $\{x_1, x_2, \dots, x_n\} \subset \mathbb{R}^3$ is a scattered point dataset and a set of function values $f_1, f_2, \dots\dots, f_n \in \mathbb{R}$. The goal consists of finding an interpolant $\mathrm{F}(\mathbf{x})/\mathbf{x} \in \mathbb{R}^3$, satisfying:

$$F(x_i) = f_i\,,\ i = 1, 2, \dots, n \tag{7}$$

RBF interpolant is defined as a linear weighted sum of radial basis functions which can be described as :

$$F(x) = \sum_{i=1}^{i=n} \beta_i(\psi\|x - x_i\|) \tag{8}$$

β_i: are the weights corresponding to each basis.
$\|.\|$: is the Euclidean norm on $\mathbb{R}^3$.

By applying condition (7),

$$\sum_{i=1}^{i=n} \beta_i(\psi\|x - x_i\|) = f_j\,, \qquad for\ j = 1,2, \dots\dots, n \tag{9}$$

The equation (9) can be expressed in matrix form

$$\Psi_{\psi,x}\beta = f \tag{10}$$

With

$$\Psi_{\psi,x} = \left[\psi\left\|x_i - x_j\right\|\right]_{1\le i,j\le 3n},\ f = [f_i]_{1\le i\le 3n} \text{ and } \beta = [\beta_i]_{1\le i\le 3n}$$

To obtain the weights, the linear system of equation (10) should be solved and in order to have a unique solution, $\Psi_{\psi,x}$ is required be non-singular. The solvability of the system (10) is guaranteed if $\Psi_{\psi,x}$ satisfies the condition of definite positive matrix. There are several radial basis functions that have this property. But there are some radial basis functions that cannot be positive definite or the system is badly conditioned. In such cases, a polynomial $\pi(x)$ of a certain degree $m-1$ will be added to equation (8). Therefore, the equation (8) will have a new form:

$$F(x) = \sum_{i=1}^{i=n} \beta_i(\psi\|x - x_i\|) + \sum_{j=1}^{j=m} \lambda_j\, \pi(x) \tag{11}$$

With the following constraint:

$$\sum_{i=1}^{i=n} \lambda_i\, \pi(x_i) = 0 \tag{12}$$

The implicit surface reconstruction by Radial Basis Functions can be summarized in three main steps:

- Generating Off-surface points and finding the implicit function F(x) by solving the system (10)
- Evaluating F(x) on uniform 3D grid
- Extracting a mesh from this function via the marching cube algorithm.

3.2.1.2 Moving least squares approximation

The MLS method was originally proposed by Lancaster et al. [25] in the context of functional data approximation or interpolation in $\mathbb{R}^d$. The MLS approximation of a function f is defined as follows.

Consider a set D $=\{(x_i, f_i), f(x_i) = f_i\}$ of sample points; *at each point* x_i *of* $\mathbb{R}^d$ *is associated with the value* f_i *of the function to be reconstructed.* The MLS approximation of $f(x)$ of degree m is given by the value $\tilde{g}(x)$ of the polynomial $\tilde{g}$ of degree m defined by:

$$\tilde{g} = \underset{g\in P^m[\mathbb{R}^d]}{argmin} \sum_i (f_i - g(x_i))^2 \ \phi(\|x_i - x\|) \tag{13}$$

where ϕ is a radially symmetric, positive and monotonically decreasing weight function, and $\|.\|$ is the Euclidian distance in $\mathbb{R}^d$. MLS surfaces have been introduced by Levin [12]and Alexa et al. [26] MLS surfaces are defined as an iterative projection procedure that projects any point near the surface onto the surface. Then, the surface is defined by the set of fixed points of the operator Ψ_P , such that $S = \{x \in \mathbb{R}^3, \Psi_P(x) = x\}$. Alexa and al. [26], detailed the procedure projection which can be divided into three steps:

1- Firstly, for a point r near to MLS surface, a local reference plane H is determined by minimizing the weighted sum of squares

$$\sum_i ({p_i}^T . n - d)^2 \ \phi(\|p_i - q\|). \tag{14}$$

Where n is the normal to the plane, q is the orthogonal projection of r on H, d is the distance of q from r and p_i is a point from surface S.

2- Secondly, A local polynomial approximation g is fitted through the points in the neighbourhood of r. To compute polynomial approximation g, a least squares approximation is used to minimize the equation:

$$\tilde{g} = \underset{g\in P^m[\mathbb{R}^2]}{argmin} \sum_i (f_i - g(u_i, v_i))^2 \ \phi(\|p_i - q\|). \tag{15}$$

Where (u_i, v_i, fi) are the coordinates of p_i in the coordinate space of the reference domain H.

3-The projection of r onto the surface is defined as:

$$r' = \Psi_P(r) = q + g(0,0). n. \tag{16}$$

The normal to the MLS surface is then defined thanks to the derivatives $\frac{\partial g(0,0}{\partial u}$ and $\frac{\partial g(0,0}{\partial v}$ of the polynomial g.

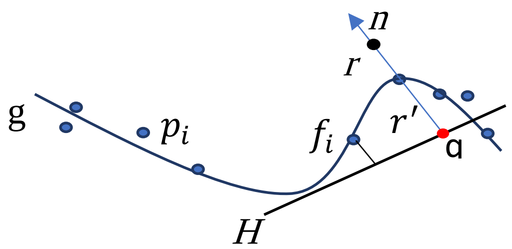


Fig. 6 Polynomial locally approximating the point cloud

3.2.2 Poisson surface reconstruction

The Poisson method developed in the paper of Kazhdan et al. [7], is both global and local. It is global because it considers all points, and local since it is based on a hierarchy of basic functions with a support compact which makes the Poisson equation a sparse linear system. the Poisson technique is based on finding 3D indicator function χ_P, and then extracting the iso-surface by marching cube algorithm.

3.2.3 Feature preserving point set surfaces based on non-linear kernel regression

The Feature Preserving Point Set Surfaces based on Non-Linear Kernel Regression method proposed by Oztireli et al. [8], is a reconstruction technique that combines the simplicity of implicit MLS surfaces with the power of robust statistics using kernel regression.

3.2.4 Implicit methods limitations

- Solving the linear system requires a sufficient memory.

4 Experiments and Results

4.1 Experimental environment and dataset

All calculations in these experiments were executed on Microsft windows 10 and via computer with Intel(R) Core (TM) i5-5300U CPU 2.30GHz Processor and 8 Go of memory (RAM). Programming was written by python language version 3.8 and Matlab R2020a. And we used MeshLab 64 bit version 2021.10 for the presentation and visualization of mesh quality.

In this work we will work on the cat object with 11172 3D points that we will scan using the EinScan SP scanner. The material used is shown in the following fig. 7.

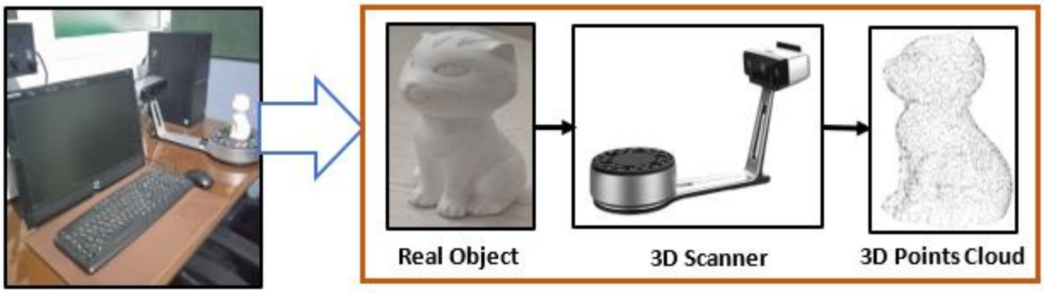


Fig. 7 Used material for object scan

4.2 Results and discussion

In this part we present the results obtained after the calculations. For this reason, we will use the point cloud of the cat model that we digitized in our laboratory. In this context, we started with the reconstruction of the surface of the cat model using the following four different reconstruction methods: alpha shape [5], ball pivoting [6], crust [4], Poisson method [7] and C. Öztireli et al. [8]. We obtain the surfaces presented in Fig. 8.

According to the results obtained, it can be clearly noticed that the surface obtained by the Crust method and Poisson method do not contain holes, and which clearly shows the details of the object. On the other hand, the method of alpha shapes and Ball Pivoting present holes and less details. These differences between these methods are due to the nature of the methods.

Alpha shape ($\alpha = 1.86$)	**Ball Pivoting ($r = 1.267$)**	**Crust**	**Poisson Depth=9, Scale=3**	**C.Öztireli et al.**

Fig. 8 Surface reconstruction using different methods

The evaluation of the surfaces produced by the different reconstruction methods is a preliminary step. This step allows us to make a comparison between the different surfaces. To make this assessment, the surface quality must be calculated. For this reason, we will use a quality criterion such as that of Frey. In fig. 9 we find the quality histogram of the surface as a function of the number of triangles which constitute the surface. There is also, the map colour which represents the distribution of triangles according to mesh quality.

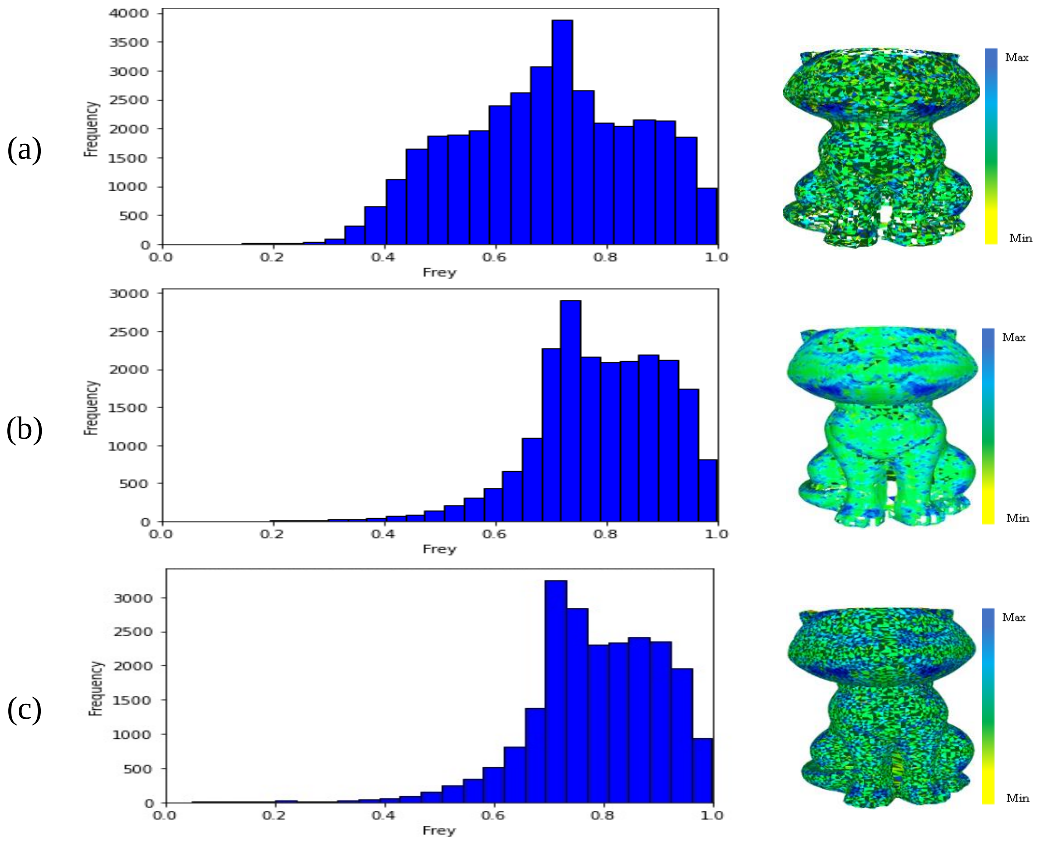
According to the histograms and the images in fig. 9, as well as table 1, it can be seen that the Ball Pivoting method and the Crust method give the best results in terms of surface quality. Moreover, statically speaking, we notice that the variance of their histograms is smaller, which explains the Frey quality results of these two methods.

In terms of surface quality, one can notice, according to the results obtained using the different combinatorial and implicit methods, that the combinatorial techniques give more compact surfaces than those obtained through of the implicit methods. Implicit techniques based on MLS present very smooth surfaces compared to those obtained using combinatorial methods.

The high quality of the surfaces obtained through combinatorial methods such as alpha shapes, Ball Pivoting, and Crust is due to the fact that these methods respect the conditions of Delaunay, during the reconstruction phase.

Table 1: Results concerning the different reconstruction methods

Methods	**Frey Quality (%)**	**Number of triangles**	**Vitesse (Triangles/s)**
Alpha shape $\alpha = 1.86$	71.297	39,698.000	16,440.702
Ball pivoting r =1.267	94.54	21,522.000	62,782.0481
Crust	94.157	22,181.000	16,808.881
Poisson Depth = 9, scale = 3	59.817	33,818.000	24,753.022
C.Öztireli et al.	59.11	12,4477.00	20,622.43



(d)

(e)

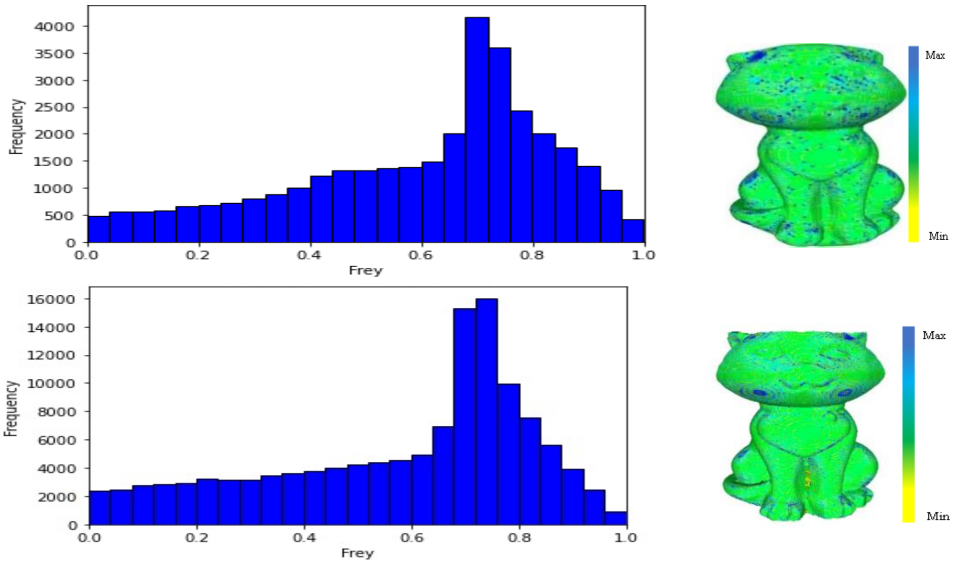


Fig. 9 Quality of mesh: a) Alpha-Shapes, b) Ball-Pivoting, c) Crust, d) Poisson, e) C.Öztireli et al. method

5 Conclusion

In this paper, a comparative study was made between different methods of 3D surface reconstruction. The objective of this comparison is to facilitate the choice of the method according to some criteria. The comparison is based on the quality of the surface generated by the different reconstruction techniques and by the reconstruction speed. The mesh quality is calculated by Frey's quality. The evaluation of these methods leads to the conclusion that Ball Pivoting gives better results regarding surface quality. And, the alpha shapes technique produces good results in terms of speed.

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